Available online at www.sciencedirect.com
SCIENCE DIRECT

Journal of Sound and Vibration 296 (2006) 734–745

JOURNAL OF SOUND AND VIBRATION

www.elsevier.com/locate/jsvi

Nonlinear stochastic optimal control of offshore platforms under wave loading

M. Luo[a,b], W.Q. Zhu[a,*]

[a] *Department of Mechanics, State Key Laboratory of Fluid Power Transmission and Control, Zhejiang University, Hangzhou, 310027, People's Republic of China*
[b] *College of Information Science and Engineering, East China University of Science and Technology, 130 Meilong Road, Xuhui District, Shanghai, 200237, People's Republic of China*

Received 6 April 2004; received in revised form 2 May 2005; accepted 31 January 2006
Available online 2 June 2006

Abstract

A nonlinear stochastic optimal (NSO) control strategy for wave-excited jacket-type offshore platforms is proposed. Wave force is determined according to linearized Morison equation. Spectral density functions of water particle velocity and acceleration are approximated by some rational forms, respectively. Wave force vector is then treated as output of a linear filter driven by white noise. A set of partially averaged Itô equations for controlled modal energies are derived by applying stochastic averaging method for quasi-integrable Hamiltonian systems. Optimal control law is then determined by using stochastic dynamical programming principle. To demonstrate the effectiveness and efficiency of the proposed control strategy, performances of uncontrolled, linear quadratic Gaussian (LQG)-controlled and NSO-controlled example platforms under different sea states are evaluated. Numerical results show that the NSO controller has better control effectiveness and efficiency than the LQG controller.

1. Introduction

Active control of civil engineering structures has been investigated for more than twenty years [1]. Many control strategies and control devices have been proposed for active control of civil engineering structures. However, most studies care about structures excited by wind and earthquakes and only a few studies concern wave-excited offshore structures [2–8]. Active control of wave-excited offshore platforms has not been appropriately investigated. As platform moves far and far into deep water, extreme wave in deep water can induce large motion of offshore platform, which can threaten safety and operation of platform. To prevent fatigue damage and to protect operation and crew of platform, more attention should be paid to active control of wave-excited offshore structures.

Since dynamic loading such as earthquake, wind and wave are usually modeled as random processes, to apply stochastic optimal control theory to civil engineering structures is natural and reasonable. Recently,

*Corresponding author. Tel.: +86 571 8795 3102; fax: +86 571 8795 2651.
E-mail address: wqzhu@yahoo.com (W.Q. Zhu).

0022-460X/$ - see front matter
doi:10.1016/j.jsv.2006.01.071

based on the stochastic averaging method for quasi-Hamiltonian systems and stochastic dynamical programming principle, a nonlinear stochastic optimal control strategy was proposed by the present second author and his co-workers [9,10]. It has been applied to the control of buildings structures under wind and earthquake excitation, and proved to be more effective and efficient than linear quadratic Gaussian (LQG) control strategy [10,11]. The present paper is devoted to the application of the nonlinear stochastic optimal control strategy to wave-excited offshore structures. Although the basic control strategy is the same, the structures and loadings are different here and in Refs. [10,11]. So, the control effectiveness and efficiency are different. Besides, the control system here is assumed to be completely observable rather than partially observable as in Refs. [10,11].

To apply the nonlinear stochastic optimal strategy to wave-excited offshore structures, it is necessary to establish a set of stochastic differential equations for random wave loading, which is usually characterized by power spectral density function. An filter approaches to wave kinematics was presented [12], extended and used for active control of offshore platform subject to wave loading using H_2 controller [13]. Here this filter approach is used in the active control of offshore platform subject to wave loading using the nonlinear control strategy.

2. Model of wave load

It is assumed that wave surface elevation $\eta(t)$ is a zero-mean, stationary, Gaussian random process, characterized by its spectral density function $S_\eta(\omega)$. One of the most commonly used spectral density function of wave surface elevation is JONSWAP spectral density [14]

$$S_\eta(\omega) = \left(\frac{5H_s^2}{16\omega_0}\right)(\omega/\omega_0)^{-5}\exp\left[-\frac{5}{4}(\omega/\omega_0)^{-4}\right]\gamma^\beta, \tag{1}$$

where H_s is significant wave height; ω_0 is dominant (peak) wave frequency, γ is sharpness magnification factor

$$\beta = \exp\left[-(\omega-\omega_0)^2/2\tau^2\omega_0^2\right] \tag{2}$$

in which $\tau = 0.07$ for $\omega \le \omega_0$ and $\tau = 0.09$ for $\omega \ge \omega_0$.

In order to obtain a rational form for the spectral density function of wave force, the following approximate JONSWAP spectral density is taken [12]

$$S_\eta(\omega) \approx \frac{G(\omega/\omega_0)^4}{\left[\left((\omega/\omega_0)^2 - k_1\right)^2 + \left(c_1(\omega/\omega_0)\right)^2\right]\left[\left((\omega/\omega_0)^2 - k_2\right)^2 + \left(c_2(\omega/\omega_0)\right)^2\right]}, \tag{3}$$

where the parameters G, k_1, k_2, c_1, c_2, are determined by using a least-squares algorithm.

Following the linear wave theory, the spectral density function of horizontal water-particle velocity $\dot{u}$ at level z is

$$S_{\dot{u}\dot{u}}(\omega,z) = \left|H_{\dot{u}\eta}(\omega,z)\right|^2 S_\eta(\omega) = \omega^2\frac{\cosh^2(\kappa z)}{\sinh^2(\kappa d)}S_\eta(\omega), \tag{4}$$

where $H_{\dot{u}\eta}(\omega,z)$ is the frequency response function for transformation from wave surface elevation to horizontal water-particle velocity at level z; $\kappa = \omega^2/g$ is wave number, g is the gravity acceleration. The spectral density function of horizontal water-particle acceleration $\ddot{u}$ at level z is

$$S_{\ddot{u}\ddot{u}}(\omega,z) = \omega^2 S_{\dot{u}\dot{u}}(\omega,z). \tag{5}$$

The frequency response function in Eq. (4) can be expanded as [13]

$$\left|H_{\dot{u}\eta}(\omega,z)\right|^2 = \omega^2\frac{\cosh^2(\kappa z)}{\sinh^2(\kappa d)} = \omega^2\frac{1+\frac{1}{2}(\kappa z)^2+\frac{1}{24}(\kappa z)^4+\frac{1}{720}(\kappa z)^6+\cdots}{1+\frac{1}{6}(\kappa d)^3+\frac{1}{120}(\kappa d)^5+\frac{1}{5040}(\kappa d)^7+\cdots}, \tag{6}$$

where d is the water depth at structural site. Keeping the first two terms of Fourier expansions of numerator and denominator, respectively, of $|H_{\dot{u}\eta}(\omega,z)|^2$ in Eq. (6) and substituting $\kappa = \omega^2/g$ into Eq. (6), one obtains

$$\left|H_{\dot{u}\eta}(\omega,z)\right|^2 \approx \left(\frac{z^2}{2g^2}\omega^6+\omega^2\right)\Big/\left(1+\frac{d^3}{6g^3}\omega^6\right). \tag{7}$$

Thus, the approximate spectral density function of horizontal water-particle velocity is

$$S_{\dot{u}\dot{u}}(\omega,z)=\frac{\frac{3Ggz^2\omega_0^4}{d^3}\left(\omega^4+\frac{2g^2}{z^2}\right)\omega^6}{\left[\left(\omega^2-k_1\omega_0^2\right)^2+c_1^2\omega_0^2\omega^2\right]\times\left[\left(\omega^2-k_2\omega_0^2\right)^2+c_2^2\omega_0^2\omega^2\right]\left(\omega^6+\frac{6g^3}{d^3}\right)} \tag{8}$$

and the approximate spectral density function of horizontal water-particle acceleration is

$$S_{\ddot{u}\ddot{u}}(\omega,z)=\frac{\frac{3Ggz^2\omega_0^4}{d^3}\left(\omega^4+\frac{2g^2}{z^2}\right)\omega^8}{\left[\left(\omega^2-k_1\omega_0^2\right)^2+c_1^2\omega_0^2\omega^2\right]\times\left[\left(\omega^2-k_2\omega_0^2\right)^2+c_2^2\omega_0^2\omega^2\right]\left(\omega^6+\frac{6g^3}{d^3}\right)}. \tag{9}$$

Similarly, the approximate cross-spectral density function of horizontal water-particle velocity is

$$S_{\dot{u}\dot{u}}(\omega,z_m,z_n)=\frac{\frac{3Ggz_mz_n\omega_0^4}{d^3}\left(\omega^4+\frac{2g^2}{z_mz_n}\right)\omega^6}{\left[\left(\omega^2-k_1\omega_0^2\right)^2+c_1^2\omega_0^2\omega^2\right]\times\left[\left(\omega^2-k_2\omega_0^2\right)^2+c_2^2\omega_0^2\omega^2\right]\left(\omega^6+\frac{6g^3}{d^3}\right)} \tag{10}$$

and the approximate cross-spectral density function of horizontal water-particle acceleration is

$$S_{\ddot{u}\ddot{u}}(\omega,z_m,z_n)=\frac{\frac{3Ggz_mz_n\omega_0^4}{d^3}\left(\omega^4+\frac{2g^2}{z_mz_n}\right)\omega^8}{\left[\left(\omega^2-k_1\omega_0^2\right)^2+c_1^2\omega_0^2\omega^2\right]\times\left[\left(\omega^2-k_2\omega_0^2\right)^2+c_2^2\omega_0^2\omega^2\right]\left(\omega^6+\frac{6g^3}{d^3}\right)}. \tag{11}$$

According to the Morison equation, wave force acting on the structure at level z is of the form

$$F(z,t)=k_D\dot{u}|\dot{u}|+k_M\ddot{u}, \tag{12}$$

where

$$k_D=(1/2)\rho C_DA_p, \quad k_M=\rho C_MV_p \tag{13}$$

in which ρ is density of seawater, C_D and C_M are drag and inertia coefficients, respectively, and A_p and V_p are projected area and volume of structure at level z, respectively. The linearized Morison equation is of the form [14]

$$F(z,t)=\sqrt{8/\pi}k_D\sigma_{\dot{u}}\dot{u}+k_M\ddot{u}, \tag{14}$$

where $\sigma_{\dot{u}}=\sigma_{\dot{u}}(z)$ is the standard deviation of horizontal water-particle velocity at level z. Then the cross-spectral density of the approximate wave forces acting on the structure at z_m and z_n is

$$S_F(\omega,z_m,z_n)=k_M(z_m)k_M(z_n)S_{\ddot{u}\ddot{u}}(\omega,z_m,z_n)+\frac{8}{\pi}k_D(z_m)k_D(z_n)\sigma_{\dot{u}_m}\sigma_{\dot{u}_n}S_{\dot{u}\dot{u}}(\omega,z_m,z_n), \tag{15}$$

where $S_{\ddot{u}\ddot{u}}(\omega,z_m,z_n)$ and $S_{\dot{u}\dot{u}}(\omega,z_m,z_n)$ are the cross-spectral densities of horizontal water-particle velocity and acceleration.

Suppose that the offshore structure is simplified as a lumped mass system with N lumped masses. The N-dimensional wave force vector $\mathbf{F}(t)$, where $\mathbf{F}(t)=[F_N(t),F_{N-1}(t),\cdots,F_1(t)]^{\mathrm{T}}$, acting on N lumped mass with spectral density matrix $\mathbf{S}(\omega)$ obtaining from discreting Eq. (15) can be modeled as the output of the following linear filter with a unit intensity Gaussian white noise process $w(t)$ as an input

$$\mathbf{F}(t)=\mathbf{C}_f\mathbf{y}, \quad \dot{\mathbf{y}}=\mathbf{A}_f\mathbf{y}+\mathbf{B}_fw(t), \tag{16}$$

where

$$\mathbf{A}_f = \begin{bmatrix} 0 & 1 & 0 & \cdots & & \cdots & 0 \\ 0 & 0 & 1 & & & & \vdots \\ 0 & \vdots & & 1 & & & \\ 0 & & & & 1 & & \vdots \\ 0 & \vdots & & & & 1 & 0 \\ 0 & 0 & \cdots & & \cdots & 0 & 1 \\ -a_0 & -a_1 & -a_2 & -a_3 & -a_4 & -a_5 & -a_6 \end{bmatrix}, \quad \mathbf{B}_f = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}_{7\times 1}, \tag{17}$$

$$\mathbf{C}_f = \left[\mathbf{0},\mathbf{0},\mathbf{0},\sqrt{\tfrac{8}{\pi}}\boldsymbol{\sigma}_{\dot{\mathbf{u}}}\mathbf{k}_D\mathbf{b}_0,\left(\sqrt{\tfrac{8}{\pi}}\boldsymbol{\sigma}_{\dot{\mathbf{u}}}\mathbf{k}_D\mathbf{b}_1+\mathbf{k}_M\mathbf{b}_0\right),\left(\sqrt{\tfrac{8}{\pi}}\boldsymbol{\sigma}_{\dot{\mathbf{u}}}\mathbf{k}_D\mathbf{b}_2+\mathbf{k}_M\mathbf{b}_1\right),\mathbf{k}_M\mathbf{b}_2\right]_{N\times 7},$$

$$a_0 = k_1k_2\omega_0^4\sqrt{\frac{6g^3}{d^3}}, \quad a_1 = (c_1k_2+c_2k_1)\omega_0^3\sqrt{\frac{6g^3}{d^3}}, \quad a_2 = (k_2+c_1c_2+k_1)\omega_0^2\sqrt{\frac{6g^3}{d^3}},$$

$$a_3 = k_1k_2\omega_0^4+(c_2+c_1)\omega_0\sqrt{\frac{6g^3}{d^3}}, \quad a_4 = (c_1k_2+c_2k_1)\omega_0^3+\sqrt{\frac{6g^3}{d^3}},$$

$$a_5 = (k_2+c_1c_2+k_1)\omega_0^2, a_6 = (c_2+c_1)\omega_0,$$

$$\boldsymbol{\sigma}_{\dot{\mathbf{u}}} = \mathrm{diag}(\sigma_{\dot{u}_{z_N}},\sigma_{\dot{u}_{z_{N-1}}},\ldots\sigma_{\dot{u}_{z_1}}),$$

$$\mathbf{k}_D = \mathrm{diag}(k_{D_{Z_N}},k_{D_{Z_{N-1}}},\ldots,k_{D_{Z_1}}), \quad \mathbf{k}_M = \mathrm{diag}(k_{M_{Z_N}},k_{M_{Z_{N-1}}},\ldots,k_{M_{Z_1}}),$$

$$\mathbf{b}_0 = \sqrt{\frac{3Gg\omega_0^4}{d^3}}\begin{bmatrix} z_N \\ z_{N-1} \\ \vdots \\ z_1 \end{bmatrix}, \quad \mathbf{b}_1 = \sqrt[4]{\frac{72G^2g^4\omega_0^8}{d^6}}\begin{bmatrix} \sqrt{z_N} \\ \sqrt{z_{N-1}} \\ \vdots \\ \sqrt{z_1} \end{bmatrix}, \quad \mathbf{b}_2 = \sqrt{\frac{6Gg^3\omega_0^4}{d^3}}\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{N\times 1}, \tag{18}$$

where diag denotes diagonal matrix. Note that, since the highest power of ω in the denominator of approximate rational spectral density function of linearized wave force is 14, the dimension of matrix $\mathbf{A}_f$ is 7×7.

3. Equation of controlled system

The simplified controlled system equation is of the form

$$\mathbf{M}\ddot{\mathbf{X}}+\mathbf{C}\dot{\mathbf{X}}+\mathbf{K}\mathbf{X} = \mathbf{F}(t)+\mathbf{P}\mathbf{U}, \quad \mathbf{X}(0) = \mathbf{X}_0, \tag{19}$$

where $\mathbf{X},\dot{\mathbf{X}},\ddot{\mathbf{X}}$ are N-dimensional vectors of structural displacements, velocities and accelerations, respectively.$\mathbf{M}$,$\mathbf{C}$ and $\mathbf{K}$ are mass, damping and stiffness matrices of the system, respectively; $\mathbf{U} = [U_1,U_2,\ldots,U_m]^{\mathrm{T}}$ is m-dimensional vector of control forces produced by m control devices; $\mathbf{P}$ is $N\times m$ control device placement matrix. Introduce model transform

$$\mathbf{X} = \boldsymbol{\Phi}\mathbf{Q}, \tag{20}$$

where $\mathbf{Q} = [Q_1,Q_2,\cdots,Q_n]^{\mathrm{T}}$ is the dominant modal displacement vector, and $\boldsymbol{\Phi}$ is a $N\times n$ dominant modal matrix consist of n dominant modal vectors. The equation for the n dominating modes can be written as

$$\ddot{\mathbf{Q}}+2\boldsymbol{\zeta}\boldsymbol{\Omega}\dot{\mathbf{Q}}+\boldsymbol{\Omega}^2\mathbf{Q} = \boldsymbol{\Phi}^{\mathrm{T}}\mathbf{F}(t)+\boldsymbol{\Phi}^{\mathrm{T}}\mathbf{P}\mathbf{U}, \quad \mathbf{Q}(0) = \mathbf{Q}_0, \tag{21}$$

where $2\boldsymbol{\zeta}\boldsymbol{\Omega} = \boldsymbol{\Phi}^{\mathrm{T}}\mathbf{C}\boldsymbol{\Phi};\boldsymbol{\Omega}^2 = \mathrm{diag}(\omega_i^2) = \boldsymbol{\Phi}^{\mathrm{T}}\mathbf{K}\boldsymbol{\Phi}$. ω_i and ζ_i are the frequency and damping coefficient of the ith mode, respectively.

Eqs. (16) and (21) can be combined and converted into the following Itô stochastic differential equation

$$\mathrm{d}\mathbf{Z} = (\mathbf{AZ} + \mathbf{BU}) + \mathbf{C}\,\mathrm{d}\tilde{B}(t), \quad \mathbf{Z}(0) = \mathbf{Z}_0, \tag{22}$$

where $\mathbf{Z} = [\mathbf{Q}^{\mathrm{T}}, \dot{\mathbf{Q}}^{\mathrm{T}}, \mathbf{y}^{\mathrm{T}}, \dot{\mathbf{y}}^{\mathrm{T}}]^{\mathrm{T}}$; $\tilde{B}(t)$ is a standard Wiener process, $\mathbf{Z}_0$ is a Gaussian random vector representing the initial state of the system, which is independent of $\tilde{B}(t)$,

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ -\boldsymbol{\Omega}^2 & -2\boldsymbol{\zeta}\boldsymbol{\Omega} & \mathbf{C}_f & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_f & \mathbf{0} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\Phi}^{\mathrm{T}}\mathbf{P} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{B}_f \\ \mathbf{0} \end{bmatrix}. \tag{23}$$

4. Stochastic averaging

To simplify the equation of controlled system and reduce its dimension, the stochastic averaging method for quasi-integrable Hamiltonian system [15] is applied to the equations for $\mathbf{Q}$ and $\dot{\mathbf{Q}}$ in Eq. (22) except the control force terms. Then the obtained partially averaged Itô stochastic differential equations for the n dominant modal energies are

$$\mathrm{d}H_i = \left[m_i(H_i) + <(\partial H_i/\partial \dot{Q}_i)\Phi_{ik}^{\mathrm{T}} P_{kr} U_r>\right]\mathrm{d}t + \sigma_i(H_i)\,\mathrm{d}\bar{B}_i(t), \quad i = 1, 2, \ldots, n, \tag{24}$$

where $<\cdot>$ denotes averaging operation with respect to time t;

$$H_i = (\dot{Q}_i^2 + \omega_i^2 Q_i^2)/2, \quad i = 1, 2, \ldots, n \tag{25}$$

denotes the energy of the ith mode; $\bar{B}_i(t)$ are standard Wiener processes;

$$\begin{aligned} m_i(H_i) &= -2\zeta_i\omega_i H_i + \Phi_{il}^{\mathrm{T}}\Phi_{ki} S_{lk}(\omega_i), \\ \sigma_i^2(H_i) &= 2H_i \Phi_{il}^{\mathrm{T}}\Phi_{ki} S_{lk}(\omega_i); \end{aligned} \tag{26}$$

$S_{lk}(\omega_i) = S_{F_l F_k}(\omega_i)$is the l, kth element of spectral density matrix $\mathbf{S}(\omega)$ evaluated at ω_i.

5. Optimal control law

Eq. (24) implies that H_i are controlled diffusion processes. The objective of stochastic optimal control is to seek the optimal feedback control $\mathbf{U}^*$ for minimizing the following semi-infinite time-interval performance index

$$J(\mathbf{U}) = \lim_{t_f \to \infty} \frac{1}{t_f} \int_0^{t_f} L(\mathbf{H}(t), <\mathbf{U}(t)>)\,\mathrm{d}t, \tag{27}$$

where $\mathrm{L}(\mathbf{H}(t), <\mathbf{U}(t)>)$ is the so-called cost function. Based on the stochastic dynamical programming principle, the following dynamical programming equation can be established [9]

$$\lambda = \min_{\mathbf{U}} \left\{ L(\mathbf{H}(t), <\mathbf{U}(t)>) + \left(\frac{\partial V}{\partial \mathbf{H}}\right)^{\mathrm{T}} \left[\mathbf{m}(\mathbf{H}) + <\frac{\partial \mathbf{H}}{\partial \dot{\mathbf{Q}}}\boldsymbol{\Phi}^{\mathrm{T}}\mathbf{PU}>\right] + \frac{1}{2}\mathrm{tr}\left[\frac{\partial^2 V}{\partial \mathbf{H}^2}\boldsymbol{\sigma}(\mathbf{H})\boldsymbol{\sigma}^{\mathrm{T}}(\mathbf{H})\right] \right\}, \tag{28}$$

where

$$\lambda = \lim_{t_f \to \infty} \frac{1}{t_f} \int_0^{t_f} [L(\mathbf{H}(t), <\mathbf{U}^*(t)>]\,\mathrm{d}t \tag{29}$$

is the optimal average cost, V is the value function and $\mathbf{U}^*$ is the optimal control force vector.

The expression for $\mathbf{U}^*$ can be obtained from minimizing the right-hand side of Eq. (28) with respect to $\mathbf{U}$. Suppose that the cost function L is of the form

$$L = g(\mathbf{H}) + <\mathbf{U}^{\mathrm{T}}\mathbf{RU}>, \tag{30}$$

where $\mathbf{R}$ is a $n \times n$ positive-define matrix. Then $\mathbf{U}^*$ is of the following form:

$$\mathbf{U}^* = -\frac{1}{2}\mathbf{R}^{-1}\mathbf{P}^{\mathrm{T}}\mathbf{\Phi}(\partial \mathbf{H}/\partial \dot{\mathbf{Q}})^{\mathrm{T}}(\partial V/\partial \mathbf{H}) \tag{31}$$

Substituting Eq. (31) into Eq. (28) and completing averaging $\mathbf{Q}$ and $\dot{\mathbf{Q}}$ with respective to t yield the final dynamical programming equation for semi-infinite time-interval control problem

$$\lambda = g(\mathbf{H}) + \sum_{i=1}^{n}\left[m_i(H_i)\frac{\partial V}{\partial H_i} - \frac{1}{4}\Delta_{ii}H_i\left(\frac{\partial V}{\partial H_i}\right)^2 + \frac{1}{2}\sigma_i^2(H_i)\frac{\partial^2 V}{\partial H_i^2}\right], \tag{32}$$

where $\Delta_{ii} = [\mathbf{\Phi}^{\mathrm{T}}\mathbf{P}\mathbf{R}^{-1}\mathbf{P}^{\mathrm{T}}\mathbf{\Phi}]_{ii}$.

$\partial V/\partial \mathbf{H}$ can be obtained by solving the final dynamical programming equation (32) and the optimal control $\mathbf{U}^*$ can be obtained by substituting the resultant $\partial V/\partial \mathbf{H}$ into the Eq. (31).

6. Response prediction of controlled structure

Averaging$(\partial H_i/\partial Q_i)\Phi_{ik}^{\mathrm{T}}P_{kr}U_r^*)$ and then substituting it into equation (24) to replace $(\partial H_i/\partial \dot{Q}_i)\Phi_{ik}^{T}P_{kr}U_r$, one obtains the completely averaged Itô equation for H_i

$$\mathrm{d}H_i = \left[m_i(H_i) + m_i^{(U)}(H_i)\right]\mathrm{d}t + \sigma_i(H_i)\,\mathrm{d}\bar{B}_i(t), \tag{33}$$

where

$$m_i^{(U)}(H_i) = -\frac{1}{2}\Phi_{ik}^{\mathrm{T}}P_{kr}R_{rs}^{-1}P_{sl}^{\mathrm{T}}\Phi_{li}H_i\frac{\partial V}{\partial H_i}. \tag{34}$$

The stationary probability density $p(\mathbf{H})$ can be obtained from solving the reduced Fokker-plank-kolmogorov (FPK) equation associated with Itô Eq. (33). It is of the following form

$$p(\mathbf{H}) = C_0\exp\left\{-\int_0^{\mathbf{H}}\sum_{i=1}^{n}\left[\left(D_i + E_i\frac{\partial V}{\partial H_i}\right)\mathrm{d}H_i\right]\right\}, \tag{35}$$

where C_0 is a normalization constant,

$$\begin{aligned} D_i &= 4\zeta_i\omega_i/\Phi_{il}^{\mathrm{T}}\Phi_{ki}S_{lk}(\omega_i), \\ E_i &= \Phi_{ik}^{\mathrm{T}}P_{kr}R_{rs}^{-1}P_{sj}^{\mathrm{T}}\Phi_{ji}/\Phi_{il}^{\mathrm{T}}\Phi_{ki}S_{lk}(\omega_i). \end{aligned} \tag{36}$$

The stationary probability density of the modal displacement and velocity of the controlled structure is then [15]

$$p(\mathbf{Q},\dot{\mathbf{Q}}) = [p(\mathbf{H})/T(\mathbf{H})]\big|_{H_i=(\dot{Q}_i^2+\omega^2Q_i^2)/2}, \tag{37}$$

where

$$T(\mathbf{H}) = \prod_{i=1}^{n}\oint\frac{\mathrm{d}Q_i}{\sqrt{2H_i - \omega_i^2Q_i^2}} = \prod_{i=1}^{n}\left(\frac{2\pi}{\omega_i}\right). \tag{38}$$

The mean square values of the modal displacement and velocity of the controlled structure are

$$\begin{aligned} E\left[Q_i^2\right] &= \int_{-\infty}^{\infty}Q_i^2p(\mathbf{Q},\dot{\mathbf{Q}})\,\mathrm{d}\mathbf{Q}\,\mathrm{d}\dot{\mathbf{Q}} = \frac{1}{\omega_i^2}\int_0^{\infty}H_ip(\mathbf{H})\,\mathrm{d}\mathbf{H}, \\ E\left[\dot{Q}_i^2\right] &= \int_{-\infty}^{\infty}\dot{Q}_i^2p(\mathbf{Q},\dot{\mathbf{Q}})\,\mathrm{d}\mathbf{Q}\,\mathrm{d}\dot{\mathbf{Q}} = \int_0^{\infty}H_ip(\mathbf{H})\,\mathrm{d}\mathbf{H}. \end{aligned} \tag{39}$$

Finally, the mean square displacement and acceleration of each floor are obtained from Eq. (39) by using modal transformation (20) as follows:

$$E\left[X_i^2\right] = E\left[\left(\sum_{j=1}^{n} \Phi_{ij} Q_j\right)^2\right],$$

$$E\left[\ddot{X}_i^2\right] = E\left[\left(\sum_{j=1}^{n} \Phi_{ij}\left(\omega_j^2 Q_j + 2\zeta_i\omega_i\dot{Q}_j - \left(\mathbf{\Phi}^{\mathrm{T}}\mathbf{F}(t) + \mathbf{\Phi}^{\mathrm{T}}\mathbf{PU}\right)_j^*\right)\right)^2\right]. \tag{40}$$

The response statistics of the uncontrolled structure can be obtained in a similar way by letting control force vanish.

7. Performance criteria

The main purpose of vibration control of wave-excited offshore platforms is to reduce displacement to prevent fatigue damage, and to reduce deck acceleration to protect operations and crew of offshore platforms. So, to evaluate a control strategy, the control effectiveness and efficiency of the displacements and deck acceleration are considered. Control effectiveness is defined as [9]

$$K = (\sigma_u - \sigma_c)/\sigma_u, \tag{41}$$

where σ stands for the standard deviation of displacement or deck acceleration; subscripts u and c denote uncontrolled and controlled offshore platforms, respectively. Control efficiency is defined as

$$\mu = K/\sigma_F, \tag{42}$$

where $\sigma_F = \sum_{i=1}^{m}\sqrt{E\left[U_i^{*2}\right]}/\tilde{G}$ is the standard deviation of control force, normalized by total weight $\tilde{G}$ of offshore platform. It is seen from the definitions that the larger K and μ are, the better the control strategy is.

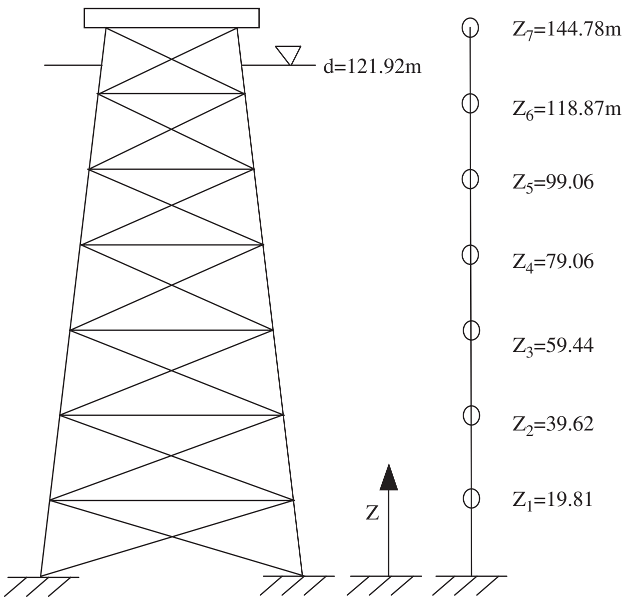

Fig. 1. Model of jacket-type offshore platform used for numerical example.

8. Numerical example

A simplified jacket-type platform with 7 lumped masses is taken as an example (see Fig. 1). The height of the platform is 144.78 m. The mass, stiffness and damping matrices are

$$\mathbf{M} = \mathrm{diag}\{4818 \quad 1475 \quad 1302 \quad 1533 \quad 1840 \quad 2205 \quad 3738\} \times 10^3\,\mathrm{kg},$$

$$\mathbf{K} = \begin{bmatrix} 344 & -289 & 17 & -1 & 26 & 4 & 16 \\ & 670 & -366 & 4 & -33 & 23 & -10 \\ & & 725 & -368 & 14 & -22 & 9 \\ & & & 777 & -410 & 23 & -23 \\ & & & & 879 & 491 & 56 \\ & \mathrm{sym} & & & & 1005 & -595 \\ & & & & & & 1344 \end{bmatrix} \times 10^3\,\mathrm{kN/m},$$

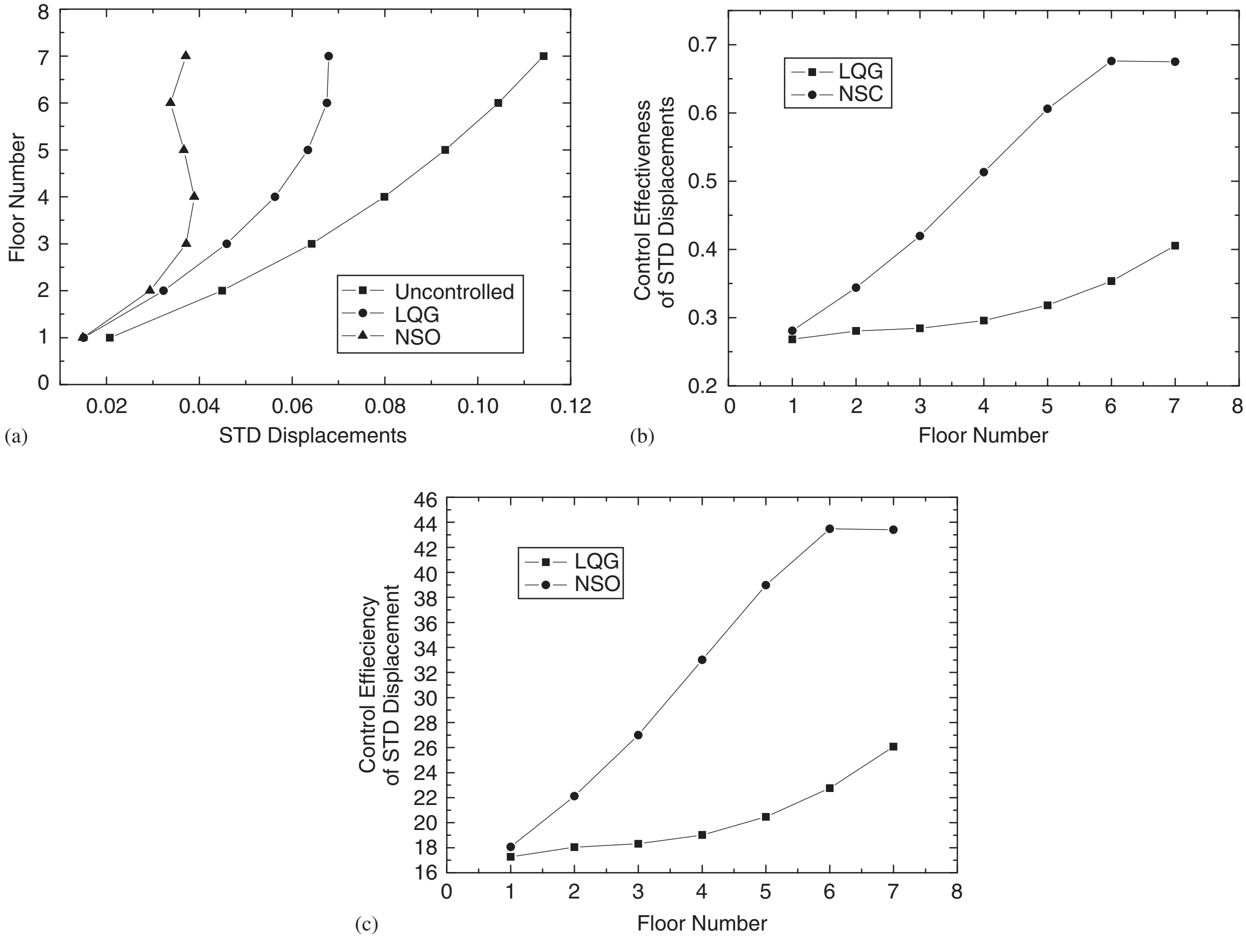

Fig. 2. (a) STD displacements of uncontrolled, NSO- and LQG-controlled offshore platforms; (b) control effectiveness and (c) control efficiency of NSO and LQG control strategies. $T = 1.5\,\mathrm{s}$, $\mathbf{R} = 10^{-7}\mathbf{I}_7$, $\mathbf{s}_1 = [1, 1, 1, 1, 1, 1, 1]^{\mathrm{T}}$ for NSO and $\mathbf{R}_l = 1$, $\mathbf{Q}_l = 10^{-7}\mathbf{I}_{14}$ for LQG.

$$\mathbf{C} = \begin{bmatrix} 2522 & -1244 & -263 & \text{-}135 & 9 & 18 & -59 \\ & 2892 & -936 & -188 & -144 & 7 & -30 \\ & & -2808 & -920 & -161 & -106 & -22 \\ & & & 3142 & -1072 & -161 & -113 \\ & & & & 3652 & -1292 & -122 \\ & \text{sym} & & & & 4311 & -1605 \\ & & & & & & 6769 \end{bmatrix} \times 10^3\,\mathrm{N\,s/m}. \tag{43}$$

The natural frequencies of this platform are 0.7432, 1.2497, 2.1678, 2.9563, 3.6921, 4.3649, 4.9773 (Hz), and the modal damping ratios are 5% for all modes.

The water depth d is 121.92 m. The vertical coordinates of lumped masses are

$$\mathbf{z} = [z_7, z_6, \cdots, z_1]^{\mathrm{T}} = [144.78, 118.87, 99.06, 79.06, 59.44, 39.62, 19.81]^{\mathrm{T}}\ \mathrm{m}; \tag{44}$$

the drag coefficient matrix is

$$\mathbf{k}_D = \mathrm{diag}\{0, 790, 676, 719, 757, 828, 1557\} \times 10^3\ \mathrm{kg/m}; \tag{45}$$

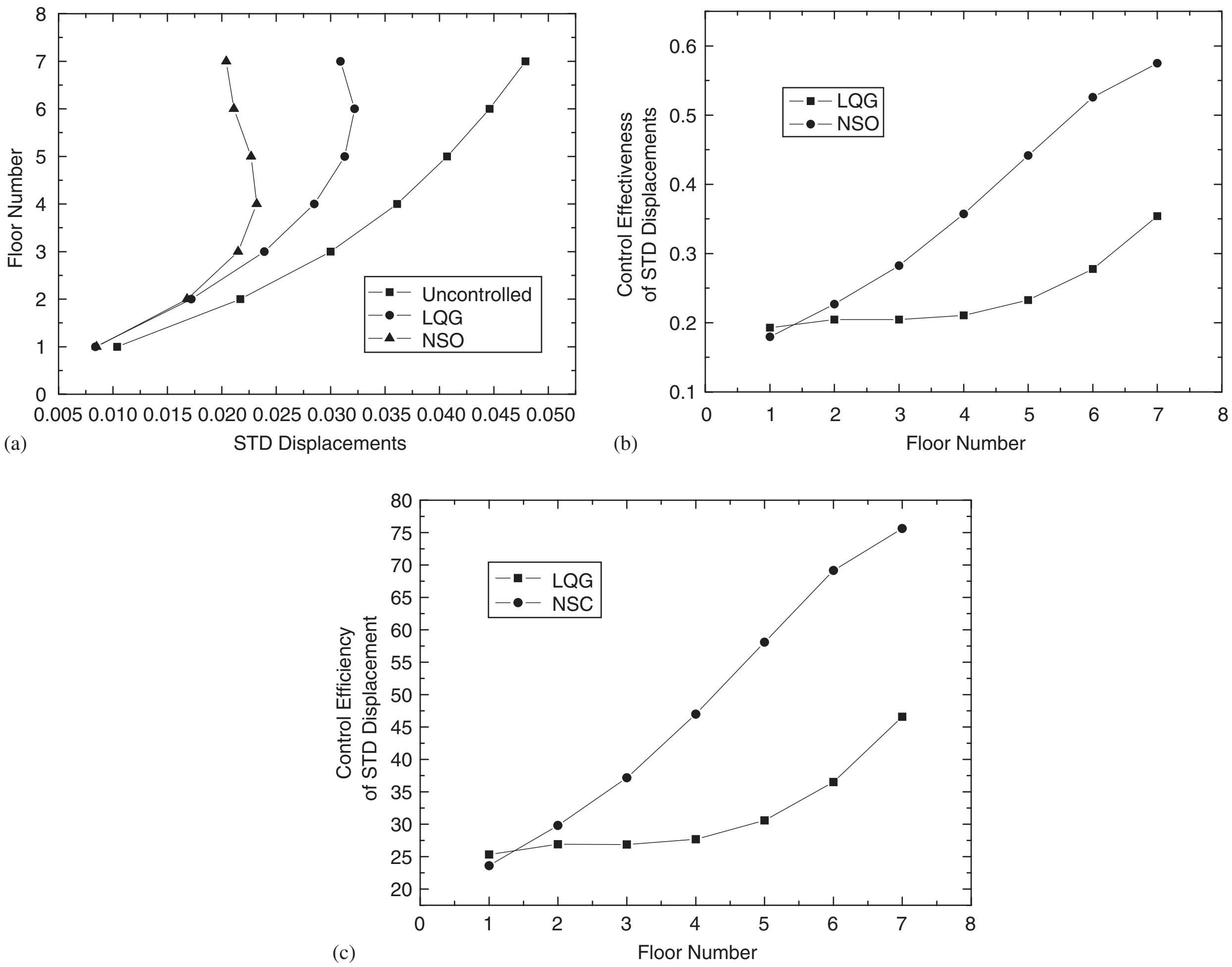

Fig. 3. (a) STD displacements of uncontrolled, NSO- and LQG-controlled offshore platforms; (b) control effectiveness and (c) control efficiency of NSO and LQG control strategies. $T = 2.5\,\mathrm{s}$, $\mathbf{R} = 10^{-7}\mathbf{I}_7$, $\mathbf{s}_1 = [1, 1, 1, 1, 1, 1, 1]^{\mathrm{T}}$ for NSO and $\mathbf{R}_l = 1$, $\mathbf{Q}_l = 10^{-7}\mathbf{I}_{14}$ for LQG.

and the inertia coefficient matrix is

$$\mathbf{k}_M = \mathrm{diag}\{0, 1744, 1674, 1939, 2563, 3150, 6735\} \times 10^3\,\mathrm{kg}. \tag{46}$$

A JONSWAP spectrum with significant wave height $H_s = 10\,\mathrm{m}$, sharpness magnification factor $\gamma = 3.0$ and periods $T = 1.5$, 2.5, 3.5 (s) is used to characterize the sea states. The parameters for the approximate JONSWAP spectrum are $G = 23.72$, $c_1 = 0.2$, $c_2 = 2.32$, $k_1 = 0.97$ and $k_2 = 0.44$.

It is difficult to obtain the exactly analytical solution to Eq. (32). However, it is possible to obtain its approximately analytical solution. For example, if

$$g(\mathbf{H}) = s_0 + \sum_{i=1}^{7} H_i(s_{1i} + s_{2i}H_i + s_{3i}H_i^2) + \sum_{i,j=1;i\neq j}^{7} s_{bij}H_iH_j + 0(H_iH_jH_k) \tag{47}$$

then a polynomial solution

$$V(\hat{\tilde{\mathbf{H}}}) = \sum_{i=1}^{7} \hat{H}_i(p_{1i} + p_{2i}\hat{H}_i) + \sum_{i,j=1;i\neq j}^{7} p_{bij}\hat{H}_i\hat{H}_j \tag{48}$$

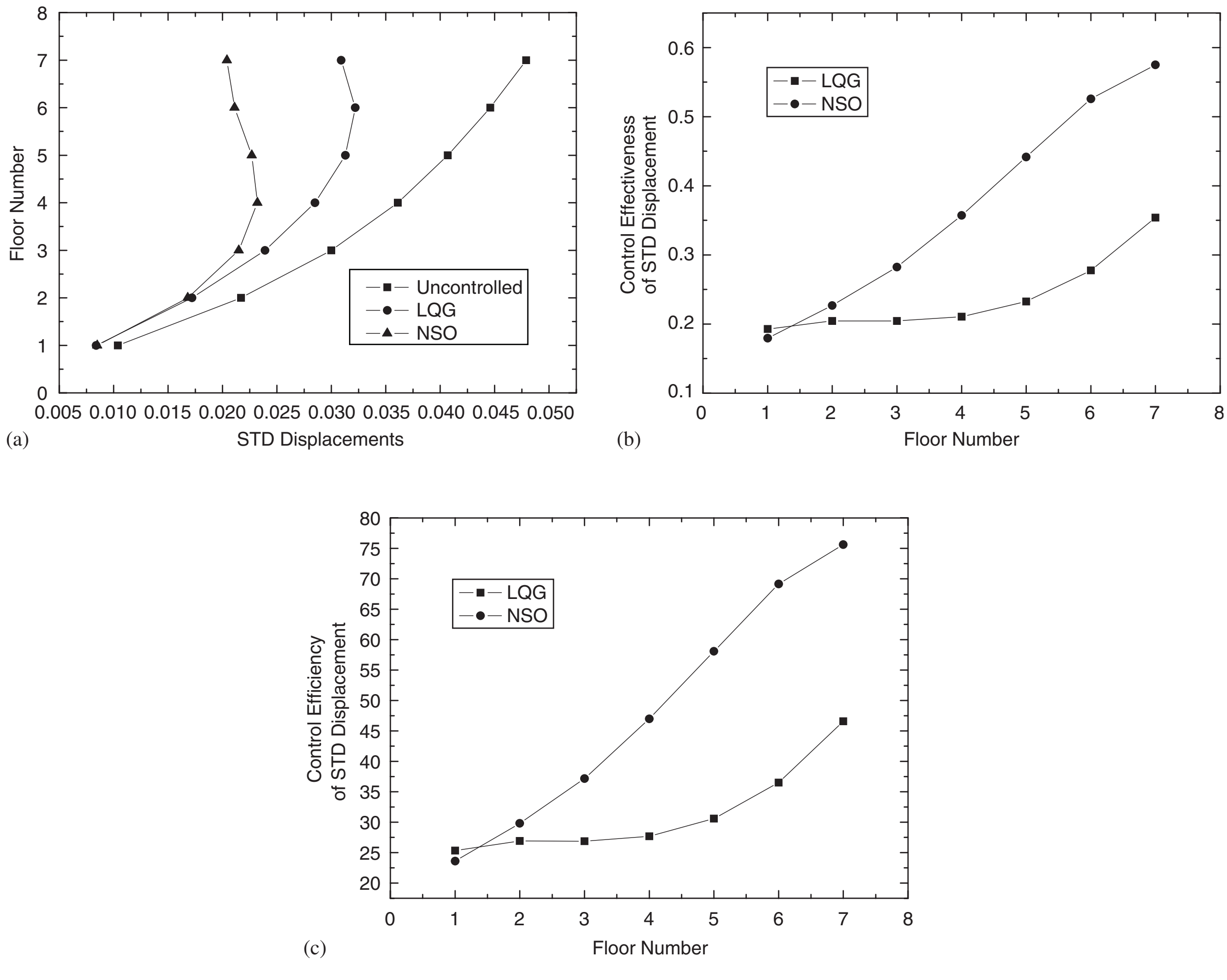

Fig. 4. (a) STD displacements of uncontrolled, NSO- and LQG-controlled offshore platforms; (b) control effectiveness and (c) control efficiency of NSO and LQG control strategies. $T = 3.5\,\mathrm{s}$, $\mathbf{R} = 10^{-7}\mathbf{I}_7$, $\mathbf{s}_1 = [1, 1, 1, 1, 1, 1, 1]^{\mathrm{T}}$ for NSO and $\mathbf{R}_l = 1$, $\mathbf{Q}_l = 10^{-7}\mathbf{I}_{14}$ for LQG.

Table 1
Standard deviation of deck acceleration and control force

Sea states	STD deck acceleration σ_a (m^2/s)			Control effectiveness K (%)		Control efficiency μ	
	Uncontrolled	NSO	LQG	NSO	LQG	NSO	LQG
$T = 1.5$	2.8004	1.2287	1.7941	56.12	35.93	36.10	23.11
$T = 2.5$	1.1950	0.5802	0.7999	51.45	33.06	67.68	43.49
$T = 3.5$	0.7285	0.3442	0.4839	52.75	33.58	110.14	70.11

to Eq. (32) may be obtained. Note that only some coefficients in Eq. (47) can be predetermined. The other coefficients in Eq. (47) and the coefficients in Eq. (48) can be determined by substituting them into dynamical programming Eq. (32).

Numerical results by using LQG control strategy are also obtained. Rewriting system Eq. (21) as system state equation

$$\dot{\mathbf{Z}}(t) = \mathbf{A}_l\mathbf{Z}(t) + \mathbf{B}_l\mathbf{U} + \mathbf{C}_l\mathbf{F}, \tag{49}$$

where $\mathbf{Z}(t) = [\mathbf{Q}^{\mathrm{T}}, \dot{\mathbf{Q}}^{\mathrm{T}}]^{\mathrm{T}}$;

$$\mathbf{A}_l = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\boldsymbol{\Omega}^2 & -2\zeta\boldsymbol{\Omega} \end{bmatrix}, \quad \mathbf{B}_l = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\Phi}^{\mathrm{T}}\mathbf{P} \end{bmatrix}, \quad \mathbf{C}_l = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\Phi}^{\mathrm{T}} \end{bmatrix}. \tag{50}$$

Let the cost function $L(\mathbf{Z}, \mathbf{U})$ be

$$L(\mathbf{Z}, \mathbf{U}) = \mathbf{Z}^{\mathrm{T}}\mathbf{Q}_l\mathbf{Z} + \mathbf{U}^{\mathrm{T}}\mathbf{R}_l\mathbf{U}, \tag{51}$$

where $\mathbf{Q}_l$ is a semi-definite symmetric matrix and $\mathbf{R}_l$ is a positive-definite symmetric matrix. In the case of semi-infinite time-interval control with performance index

$$J_l = \lim_{T\to\infty} \frac{1}{T}\int_0^T L(\mathbf{Z}, \mathbf{U})\,\mathrm{d}t \tag{52}$$

the optimal control force can be determined easily [10].

The active control device is installed on the top floor. Numerical results of STD displacement responses of uncontrolled, LQG- and NSO-controlled offshore platforms for three different sea states are shown in Figs. 2–4. Numerical results of STD deck acceleration responses are listed in Table 1. It can be seen from these numerical results that the control effectiveness and efficiency of the proposed NSO control strategy are greater than those of LQG control strategy.

9. Conclusions

In this paper a nonlinear stochastic optimal (NSO) control strategy has been proposed for offshore platform subject to wave loading. The proposed control strategy has several advantages. By using the stochastic averaging, the dimension of control system is reduced to a half of that of the original controlled system, and thus the dimension of dynamical programming equation is also reduced by a half. Numerical results for an example offshore platform subject to three different sea states shows that the proposed control strategy is more effective and efficient than LQG control strategy. Note that this is the primary study on the application of the nonlinear stochastic strategy on wave-excited offshore structures. The more practical study considering all details including measuring and estimating system state will be conducted in future.

Acknowledgements

The work reported in this paper was supported by the National Natural Science Foundation of China under Key Grant no. 10332030 and the special Fund for Doctor Programs in the Institutions of Higher Learning of China under Grant no. 20020225092.

References

[1] G.W. Housner, L.A. Bergman, T.K. Caughey, A.G. Chassiakos, R.O. Claus, S.F. Masri, R.E. Skelton, T.T. Soong, B.F. Spencer, J.T.P. Yao, Structural control: past, present, and future, *ASCE Journal of Engineering Mechanics* 123 (1997) 897–971.

[2] S. Sirlin, C. Paliou, R.W. Longman, M. Shinozuka, E. Samaras, Active control of floating structures, *ASCE Journal of Engineering Mechanics* 112 (1986) 947–965.

[3] A.M. Reinhorn, G.D. Manolis, C.Y. Wen, Active control of inelastic structures, *ASCE Journal of Engineering Mechanics* 113 (1987) 315–333.

[4] T. Hirayama, N. Ma, Dynamic response of a very large floating structure with active pneumatic control, *Proceedings of the Seventh International Offshore and Polar Engineering Conference, Honolulu*, vol I, 1997, pp. 269–276.

[5] K. Yoshida, H. Suzuki, D. Nam, M. Hineno, S. Ishida, Active control of coupled dynamic response of TLP hull and tendon, *Proceedings of the Forth International Offshore and Polar Engineering Conference, Osaka, Japan*, 1994, pp. 10–15.

[6] M. Abdel-Rohman, Structural control of a steel jacket platform, *Structural Engineering and Mechanics* 4 (1996) 125–138.

[7] J. Suhardjo, A. Kareem, Structural control of offshore platforms, *Proceedings of the Seventh International Offshore and Polar Engineering Conference*, ISOPE-97, Honolulu, 1997.

[8] J.L. Hua, J.H. Sau-Lon, J. Christopher, H_2 active vibration control for offshore platform subjected to wave loading, *Journal of Sound and Vibration* 263 (2003) 709–724.

[9] W.Q. Zhu, Z.G. Ying, T.T. Soong, An optimal nonlinear feedback control strategy for randomly excited structural systems, *Nonlinear Dynamics* 24 (2001) 31–51.

[10] W.Q. Zhu, Z.G. Ying, Nonlinear stochastic optimal control of partially observable linear structures, *Engineering Structures* 24 (2002) 333–342.

[11] W.Q. Zhu, M. Luo, Z.G. Ying, Nonlinear stochastic optimal control of tall buildings under wind loading, *Engineering Structures* 26 (2004) 1561–1572.

[12] P.T.D. Spanos, Filter approaches to wave kinematics approximation, *Applied Ocean Research* 8 (1986) 2–7.

[13] J. Suhardjo, A. Kareem, Feedback-feedforward control of offshore platforms under random waves, *Earthquake Engineering and Structural Dynamics* 30 (2001) 213–235.

[14] T. Sarpkaya, M. Isaacson, *Mechanics of Wave forces on Offshore Structures*, Van Nostrand Reihhold, New York, 1981.

[15] W.Q. Zhu, Z.L. Huang, Y.Q. Yang, Stochastic averaging of quasi-integrable Hamiltonian systems, *ASME Journal of Applied Mechanics* 64 (1997) 975–984.